PA-003-1016002

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Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March / April – 2020

Mathematics : Paper - M - 09 (A)

(Mathematical Analysis - 2 & Abstract Algebra - 2)

Faculty Code : 003
Subject Code : 1016002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are **compulsory**.

(2) Write answer of each question in your main answer sheet.

1 (A) Answer the following questions in brief : **4**

(1) Define bounded set.

(2) Define separated sets.

(3) Define connected set.

(4) Determine whether the subset {1, 2, 3, 4, 5} of metric space R is compact or not.

(B) Attempt any **one** out of **two** : **2**

(1) Show that subsets $A = (3, 4)$ and $B = (4, 5)$ of metric space R are separated.

(2) Show that every finite subset of a metric space is compact.

(C) Attempt any **one** out of **two** : **3**

(1) State and prove Heine-Borel theorem.

(2) Prove that every singleton subset of any metric space is connected.

(D) Attempt any **one** out of **two** : **5**

(1) Prove that continuous image of compact set is compact.

(2) State and prove theorem of nested intervals.

2 (A) Answer the following questions in brief : 4

(1) Define Laplace Transform

(2) Find $L\left(e^{-t}\right)$.

(3) Find $L^{-1}\left(\frac{1}{s^2-1}\right)$.

(4) Show that $L(1) = \frac{1}{s}$, where $s > 0$.

(B) Attempt any **one** out of **two** : 2

(1) Find $L^{-1}\left(\frac{s-2}{(s-2)^2+4}\right)$.

(2) Find $L(2t + 5 \sin 3t)$.

(C) Attempt any **one** out of **two** : 3

(1) Find Laplace transform of $t^2 \; sin \, at$

(2) If $L\{f(t)\} = \overline{f}(s)$ then prove that $L\{e^{at} f(t)\} = \overline{f}(s-a)$.

(D) Attempt any **one** out of **two** : 5

(1) Find Laplace inverse of $\frac{s+2}{(s-2)^3}$.

(2) Prove that $L^{-1}\left(\frac{s}{\left(s^2+a^2\right)^2}\right) = \frac{1}{2a} t \sin at.$

3 (A) Answer the following questions in brief : 4

(1) Find $L(t\,e^t)$.

(2) Write convolution theorem.

(3) Find $L(t \sin t)$.

(4) Find $L(t^2 e^{-2t})$.

(B) Attempt any **one** out of **two** : 2

(1) If $L\{f(t)\} = \overline{f}(s)$ then prove

$$L\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}\left[\overline{f}(s)\right].$$

(2) Find $L\left(\frac{\sin t}{t}\right)$.

(C) Attempt any **one** out of **two** : **3**

(1) Prove that $L\left\{\dfrac{e^{-at}-e^{-bt}}{t}\right\}=\log\left(\dfrac{s+b}{s+a}\right)$.

(2) Prove that $L^{-1}\left(\log\left(1+\dfrac{4}{s^2}\right)\right)=\dfrac{2(1-\cos 2t)}{t}$.

(D) Attempt any **one** out of **two** : **5**

(1) Prove that $L^{-1}\left\{\dfrac{s^2-a^2}{\left(s^2+a^2\right)^2}\right\}=t\cos at$.

(2) Using convolution theorem, prove

$$L^{-1}\left\{\frac{s}{\left(s^2+4\right)^2}\right\}=\frac{t}{4}\sin 2t.$$

4 (A) Answer the following questions in brief : **4**

(1) Define Natural Mapping.

(2) Define monomorphism.

(3) If $\phi:(G,*)\to(G',\Delta)$, $\phi(x)=e',\ \forall x\in G$ is a homomorphism. Then find K_ϕ.

(4) Define Kernel of homomorphism.

(B) Attempt any **one** out of **two** : **2**

(1) Let $(G, *) = (Z, +)$ and $(G', \Delta) = (R, +)$ then show that $\phi:(G,*)\to(G',\Delta);\ \phi(x)=x$ is Homomorphism.

(2) If $\phi:(G,*)\to(G',\Delta)$ is a Homomorphism. Then $\phi(e)=e'$ where e and e' are identity elements of G and G' respectively.

(C) Attempt any **one** out of **two** : **3**

(1) If $\phi:(G,*)\to(G',\Delta)$ is a Homomorphism and if N is a normal subgroup of G then show that $\phi(N)$ is a normal subgroup of G'.

(2) Prove that a Homomorphism $\phi:(G,*)\to(G',\Delta)$ is one-one iff $k_\phi=\{e\}$.

(D) Attempt any **one** out of **two** : **5**

(1) State and prove first fundamental theorem of homomorphism

(2) If $\phi:(G,*)\to(G',\Delta)$ is a Homomorphism. Then prove that Kernel k_ϕ is a normal Subgroup of G.

5 (A) Answer the following questions in briefly : **4**

(1) Define Commutative ring

(2) If polynomial $f = (0, 1, 2, 3, 0, 0,.....)$ then find degree of f.

(3) Define Field.

(4) Define Monic polynomial.

(B) Attempt any **one** out of **two** : **2**

(1) State and prove factor theorem of polynomials

(2) If $f(x) = (3, 1, 4, 2, 0, 0....)$ and $g(x) = (1, 3, 0, 0, 0, 1....) \in R[x]$ then find $f(x) + g(x)$.

(C) Attempt any **one** out of **two** : **3**

(1) State and prove Remainder theorem of polynomials

(2) In $R[x]$, $f(x) = 4x^4 - 3x^2 + 1$ is divided by $g(x) = x^3 - 2x + 1$ then find quotient $q(x)$ and remainder $r(x)$.

(D) Attempt any **one** out of **two** : **5**

(1) Let $f, g \in D[X] - \{0\}$ then prove that $[fg] = [f] + [g]$.

(2) Prove that a field has no proper ideal.
